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Albert Einstein: His Annus Mirabilis 1905*

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Abstract

Einstein in 1905, his year of miracles, wrote five papers which mark a watershed between classical physics and modern physics. They dealt with problem of reality of atoms, theory of special relativity which overthrew Newtonian conceptions of space and time, and his revolutionary light quantum hypothesis which together with Planck's work on black body radiation started the quantum revolution. We put these discoveries in the context of that period and also indicate their later influence.

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1 Year of Physics (2005)

We are having a worldwide celebration of physics throughout this year (2005). The year 2005 has been declared the "Year of Physics" by UNESCO. It is the centenary year of the "Annus Mirabilis", ie the Miracle year, 1905, of Albert Einstein. During this year he published a set of five papers dealing with the existence of atoms, special relativity including the now famous equation $E = MC^2$ expressing the equivalence of energy content E and inertial mass M of a body, as well as on the quantum theory together with its application to the photoelectron effect. These papers mark the watershed between classical physics of Isaac Newton, Michael Faraday and John Clerk Maxwell and the modern physics. It is therefore entirely appropriate that this centenary year of the annus mirabilis be celebrated as the year of physics.

We may also mention that the decade of 1895 to 1905 was extremely rich in discoveries which established the existence of a number of phenomenon which were not explicable within classical physics. This crisis would require for its resolution a change of classical framework to that of modern physics involving relativity and quantum theory. In 1895, Roentgen discovered X-rays. In 1896 the radioactivity was discovered by Henri Becquerel and magnetic field effect on spectral lines by Pietr Zeeman. In 1897, J.J. Thompson established the existence of electrons. In 1900, Max Planck introduced quantum ideas in physics. As has been noted in this connection, the twentieth century in physics began not in the year 1900, but a full five years earlier in 1895. It is also fitting that this fruitful decade (1895–1905) was capped by the Annus Mirabilis of Einstein.

Before we proceed to discuss in detail why the year 1905 is referred to as the annus mirabilis, let us briefly recall an earlier year, 1666 which is also referred to by the same designation. It was the annus mirabilis of Isaac Newton, who came to symbolise the emergence of not only classical physics and astronomy but classical mathematics.

2 Annus Mirabilis of Isaac Newton

2.1 John Dryden

The noted restoration English poet, John Dryden, published a long poem, some 116 pages in print, in 1688 usually referred to briefly as "Annus Mirabilis". It is worth quoting the title in full which was "Annus Mirabilis, The Year of Wonders, MDCLXVI, An Historical Poem. Also A Poem on the Happy Restoration and Return of His Late Sacred Majesty, Charles the Second. Likewise A Panegyric on His Coronation Together With a Poem to My Lord Chancellor, Presented on New-Year Day 1662. And a Elegy on the Death of Kind Charles the Second By John Dryden, Esq". It celebrated the annus mirabilis, 1666, the year of wonders and some of the wonders so celebrated were survival of London from the Great Fire and the Victory of English fleet over the Dutch.

This the how the term "Annus Mirabilis" entered the English language. The term was soon appropriated by scientists to refer to the achievements of Isaac Newton during that year, 1666, which truly could be called "wonders" unlike "wonders" celebrated by John Dryden in his poem.

2.2 Isaac Newton

Since some of Newton's discoveries took place in the previous year, ie 1665, we should more properly speak of his Anni Mirabilis, ie years of wonders, 1665-1666. Newton was born at the village of Woolsthorpe on christmas day 1642. Coincidentally it was the year in which Galileo died. Newton was admitted to Trinity College at the Cambridge University at the age of eighteen. Soon after he had completed his Bachelor's degree, the University was closed down due to a serious epidemic of the "Great Plagues" at Cambridge in June 1665. All the professors and students went home and Newton returned to his ancestral village of Woolsthorpe to live with his grandmother and mother. During this period of enforced idleness Newton started thinking about mathematics and scientific problem and had one of those rare concentrated burst of creativity which resulted in his laying the foundation of classical mathematics, physics and astronomy.

Some fifty year later Newton himself wrote an account of period for des Maizeaux. This account is almost invariably quoted in this connection and is as follows: "In the beginning of the year 1665 I found the Method of approximating series and the Rule for Reducing any dignity of any Binomial into such a series. The same year in May I found the method of Tangents of Gregory and Slusius, and in November had the direct method of fluxions and the next year in January had the Theory of Colours and in May following I had entrance into ye inverse method of fluxions. And the same year I began to think of gravity extending to ye orb of the Moon and ··· I deduced that the forces wch keep the Planets in their Orbs must be reciprocally as the squares of their distances from the centers about which they revolve ··· All this was in the two plague years of 1665-1666. For in those days I was in the prime of my age for invention and minded Mathematics and Philosophy more than at any other time since".

Note that Newton was a fresh undergraduate and only twentyfour years of age at the time when he discovered analysis, both differential and integral calculus, theory of colours and the law of universal gravitation.

3 Annus Mirabilis: Albert Einstein

3.1 Early life

Albert Einstein was born on 14 March 1879 at Ulm in Germany. In his childhood, in 1889, his father had presented him a pocket magnetic compass and young Einstein found the behavior of magnetic needle, which always point in a fixed north south direction, as a profound, almost mystical, experience. This gave him the idea that the physical world is subject to laws.

On July 28, 1900 he was granted Diploma by ETH (Eidgenössische Technische Hochschule) at Zurich. His grades in different courses were: Theoretical Physics - 5/6, Experimental Physics - 5/6, Astronomy - 5/6, Theory of Functions - 5.5/6 and the Diploma Paper – 4.5/6. An overall grade of 5 our of a total of 6 is quite good and might come as a surprise to those who have been exposed to the idea that Einstein was a poor student. He was indeed a poor student in his elementary school, which was more due to bad teaching, but had clearly remedied the situation by the time he was at ETH, Zurich. Not only that towards the end of that year, on December 13, 1700, he sent his first research paper for publication to the well known journal "Annalen der Physik".

He started work as a clerk at the Patents office, Bern on probation at a salary of 3500 swiss francs per year. It seems that his father had to use his personal influence with one of his friends for Albert to get this job. Soon afterwords his father died on October 10, 1902 and thus one of sources of financial security for him was cut off. On top of that his expenses increased when he married his fellow student Mileva Maric and with the birth of their first son Hans Albert on March 14, 1904, Luckily he was confirmed in his job at the Patent office on September 16, 1904.

During this period (1901-1904) when he had no proper academic appointment and subject as well to financial insecurities, it is surprising that he published five papers in Annalen der Physik. We need not comment on first two of these which deal with capillary phenomenon. But the remaining three, on his discovery of the ensemble method is statistical mechanics were rather important. Here, however, he had been scooped by Josiah W. Gibbs, who had obtained these results some what earlier.

3.2 Einstein in 1905

During this year, Einstein published five papers on statistical physics, special theory of relativity and the quantum theory apart from completing his Ph.D. dissertation.

In chronological order these were as follows:

(1) Light quantum paper: The paper "On a heuristic point of view concerning the production and transformation of light" was received by Annalen der Physik on March 18, 1905. This was published in Annalen der Physik 17, 132-148 (1905).

(2) Thesis on Molecular Sizes: The Ph.D. dissertation "On a new determination of the Molecular Dimensions" was completed on April 30, 1905. It was printed at Bern and submitted to University of Zürich on July 20, 1905. He also sent a paper based on the thesis to Annalen der Physik soon after the thesis was accepted on August 19, 1905 by the University which appeared in Annalen der Physik 19, 289-305 (1906) next year.

(3) Brownian Motion paper: The paper "On the motion of small particles suspended in liquids at rest required the Molecular Kinetic theory of heat" was received on May 11, 1905 for publication and appeared in Annalen der Physik, 17, 549-560 (1905).

(4) Special theory of relativity paper: The paper "On the electrodynamics of moving bodies" was received for publication on June 30, 1905 and appeared in Annalen der Physik 17, 891-921 (1905).

(5) $E = mc^2$ paper: The paper "Does the inertia of a body depend on its energy context" was received for publication on September 27, 1905 and appeared as Annalen der Physics 18, 639-641 (1905).

Einstein, besides the above, sent another paper on Brownian motion on December 19, 1905 to Annalen der Physik which was published next year.

We had mentioned earlier Newton's own account of his Anni Mirabilis some half a century after the event. We have an account by Einstein of his work in his annus mirabilis which was written during that very year. It occurs in letters he wrote to his friend Conrad Habicht. He, together with Einstein and Maurice Solovine, was a member of the triumvirate 'Olympia Academy', who used to meet regularly in evenings to have wide ranging intellectual discussions extending from philosophy to physics.

Einstein wrote to Habicht on May 18 or 25, 1905: "⋯ But why have you still not sent me your dissertation? ⋯ I promise you four papers in return, the first of which I might send you soon, since I will soon get complimentary reprints. The paper deals with radiation and the energy properties of light and is very revolutionary as you will see if you send me your work first. The second paper is a determination of the true sizes of atoms from the diffusion and the viscosity of dilute solutions of neutral substances. The third proves that, on the assumption of molecular theory of heat, bodies on the order of megnitude 1/1000 m.m., suspended in the liquids, must already perform an observable random motion that is produced by the thermal motion; in fact physiologists have observed (unexplained) motions of suspended small, inanimate, bodies, whose motion they designate as "Brownian Molecular motion". The fourth paper is only a rough draft at this point, and is an electrodynamics of moving bodies which employes a modification of the theory of space and time; the purely kinematical part of this paper will surely interest you. ⋯".

Einstein again wrote to him on some Friday during the period June 30, 1905 – September 22, 1905 to bring him uptodate with his later work as follows: "⋯ A consequence of the study on electrodynamics did cross my mind. Namely, the relativity principle, in association with Maxwell's fundamental equations, requires that the mass be a direct measure of the energy contained in a body; light carriers mass with it. A noticeable reduction of mass would have to take place in the case of radium. The consideration is amusing and seductive; but for all I know, God Almighty might be laughing at the whole matter and might have been leading me around by the nose".

We have excised the purely personal remarks and banter from these letters.

In the rest of the writeup we shall now discuss these contribution in somewhat more detail and provide their background and context so as to appreciate them more properly. We shall not follow the chronological order in which they were written but rather the order in which they make a transition from classical physics to modern physics. Chronologically his light quantum paper is first during 1905 but as Einstein himself remarked it is the most revolutionary. The order followed would therefore be as follows:

1. Thesis on molecular motion
2. Brownian motion paper
3. Special theory of relativity and $E = mc^2$ papers
4. Light Quantum paper.

4 Thesis on Molecular Sizes

4.1 The first attempt

The Ph.D. thesis which Einstein wrote in 1905 was not his first attempt at submitting a thesis for this degree. He first submitted a Ph.D. dissertation in November 1901. It is not known as to what the topic was. It is also not clear as to why Einstein withdrew it soon in February 1902. As he wrote to his friend Michele Besso from Bern on a Thursday (January 22(?) 1903): "I have recently decided to join the ranks of Privatdozenten, assuming, of course that I can carry through with it. On the other hand I will not go for a doctorate, because it would of little help to me, and the whole comedy has bcome boring". He however changed his mind about a doctorate degree soon afterwards.

4.2 Thesis on molecular sizes

Till 1909 ETH was not recognized as an institution allowed to grant doctoral degrees. However under a special arrangement the ETH students were permitted to submit their doctoral dissertations to University of Zürich. Einstein's thesis advisor was Alfred Kleiner who was an experimental physicist, specialised in instrumentation. He however also had a broader interest in basic physics. The thesis was dedicated to his fried Marcel Grossman, who would eventually help him with tensor calculus in his formulation of general theory of relativity in 1915.

Einstein motivated the thesis topic by pointing out that though there have been many determinations of molecular sizes till that date they all have used kinetic theory of gases. His proposed method would be the first to use phenomenon in liquids. Even though not exactly the first to do so, it was indeed the first one to give results comparable in accuracy from those obtained by much more developed kinetic theory of gases. This is qite remarkable in the absence of any available kinetic thoery of liquids. Einstein had chosen his thesis problem totally on his own as was acknowledged by his thesis advisor Kleiner.

4.3 Main results in the thesis

The thesis has two main results:

(i) Viscosity of dilute solutions

Let η and η_s be respectively the viscosity of the solvent and solution made of a solute in this liquid. Einstein's hydrodynamical investigation led to the result

$$\eta_s = \eta \left[1 + \left(\frac{5}{2} \right)^* \varphi \right]$$

where the φ is the fraction of volume occupied by the solute molecules in the solution. The solute volume fraction φ is given by

$$\varphi = \frac{4\pi}{3}(N_A a^3)(\rho_s/m_s)$$

where N_A is the Avodgadro number, a the radius of the solute molecules, ρ_s (and m_s) refer to the massdensity (and the molecular weight) of the solute.

We have put a star on the numerical factor 5/2 above as this factor was missed out in Einstein dissertation.

Einstein assumed that the solution is a dilute one and further that the solute molecules do not dissociate in the solution. He proceeds to calculate the change in flow of the solvent around solute particles, taken to be spheres, and shows that it results in an effective visiosity coefficient η_s for the solution as given above.

(ii) Diffusion coefficient of solute molecules

Einstein also established a formula for diffusion coefficient D of the solute particle in the solvent liquid. He showed that

$$D = RT/(6\pi\eta N_A a)$$

where R is the universal gas constant and T is the absolute temperature of the liquid.

We thus see that a measurement of the relative change in viscosity of a solution due to a solute allows us to determine the product $N_A a^3$ while a measurement of the diffusion coefficient allows us to know the product $N_A a$. Together these two measurements allow us a determination of both the Avogadro Number, N_A, and the size of the solute molecules, a. Einstein applied his analysis to available data on dilute solutions of sugar molecules in water and obtained

$$\begin{aligned} a &= 9.9 \times 10^{-8} cm, \\ N_A &= 2.1 \times 10^{23}. \end{aligned}$$

On using improved data, which became available soon he obtained a better value of N_A given by

$$N_A = 4.15 \times 10^{23}$$

during 1906. This was first of the three methods proposed by Einstein during 1905 for determining N_A. This clearly points to the importance Einstein attached to the problem of determining the Avogadro number.

4.4 Comments on the dissertation

As Alfred Kleiner noted in his report, dated 22-23 July, 1905, on the thesis: "The arguments and calculations to be carried out are among the most difficult ones in hydrodynamics, and only a person processing perspicacity and training in the handling of mathematical and physical problems could dere to tackle them, and it seems to me that Mr. Eination has proved that he is capable of working successfully on scientific problems; I would therefore recommend that the dissertation be accepted". He however added "since the main achievement of Einstein's thesis consists in the handling of differential equatons, and hence is mathematical in character and belongs in the domain of analytical mechanics, I would like to ask the dean also to approach my colleague Professor Burkhardt (Heinrich Burkhardt was Professor of Mathematics at University of Zurich) for an expert opinion".

His expert opinion was as follows: "At the request of my colleague, Professor Kleiner, I reviewed the dissertation of Mr. Einstein and checked the most important part of his calculations, that is, all of the places indicated by Professor Kleiner. What I checked, I found to be correct without exception, and the manner of treatment demonstrate *a thorough command of the mathematical methods, involved* ⋯". Incidently the correct factor of 5/2, in the expression for η_s was again missed out in this checking. In view of the discrepency between the experimental results on the solution viscosity of Jacques Bancelin, working in the laboratory of Jean Perrin, Einstein requested his student and collaborator, Ludwig Hopf, to check his calculations again. Hopf was successful in finally finding the missing factor of 5/2 in Einstein's expression. Hopf's correction was communicated to Perrin by Einstein on Jan 12, 1911. If this correction is used then we get the much more satisfactory value

$$N_A = 6.56 \times 10^{23}.$$

Initially this dissertation was foreshadowed by other papers of Einstein during this year. However, eventually, this is the paper of Einstein which has received the highest citation in view of it's use by molecular physicists and chemists. May be citation index is not such an infallible guide to the significance of a paper!

5 Brownian Motion

5.1 Atomic theory around the end of nineteenth century

Modern chemistry dates back to John Dalton's book "New System of Chemical Philosophy" in 1808 in which he proposed his system of a finite number of chemical elements. All the molecules were taken as composed of atoms of these chemical elements. Amedeo Avogadro in 1811 proposed that, under conditions of equal temperature and pressure, equal volumes of gases contain the same number of molecules for all gases. This number for a mole of gas was named by Jean Perrin as the Avogadro Number N_A. Avogadro's law presupposes the reality of molecules. Most chemists, however, used atomic theory in the nineteenth century as a theoretical heuristic device to bring order into the description of chemical phenomenon. They did not necessarily subscribe to their reality.

In the second half of the nineteenth century, the Kinetic theory of gases, which posited the gases to consist of moving molecules, made rapid progress. Clausius, in 1857, suggested that heat is a form of moleculer motion. John Clerk Maxwell proposed his famous distribution law for the molecular velocities in a gas. Ludwig Boltzmann gave his equation which set out to reduce all thermodynamic phenomenon to mechanical description using molecules. These developments in the kinetic theory of gases gave a big boost to the atoms being real entities.

At the end of nineteenth century most physicists and chemists thus either believed in the reality of molecules or at least were willing to use them as heuristics. In view of the fact that all the evidence for the atoms was indirect, as atoms were not directly seen, there was still a small but powerful opposition to the idea of their reality. The great physical chemist Ostwald, as well as George Helm, regarded atoms to be mathematical constructs. The situation in regard to atoms was similar to that of "quarks" as constituents of matter in twentieth century. Ostwald had his own program, ‘Energetics’, in which the prime ontological entity was energy. Max Planck also was of that persuasion at that time since he regarded laws of thermodynamics to be absolute laws. While in Boltzmann’s atomic view the second law of thermodynamics, regarding entropy, was only statistical in nature and not absolute. Even the great physicist and philosopher Ernst Mach was opposed to the reality of the atoms in view of “positivist” slant of his philosophy. In 1905 Einstein made a decisive impact on this debate through his paper on Brownian Motion.

5.2 Einstein’s contribution

In his paper on Brownian motion, Einstein investigated the random motions executed by visible, but very small, particles in a liquid. The visible random motion of these particles was taken to arise from their being buffeted by the incessent motion of the invisible liquid molecules. His hope was that such a study would be convincing enough about the reality of the underlying molecules of the liquid. As he noted “It will be shown in this paper that according to molecular – kinetic theory of heat, bodies of a microscopically visible size suspended in liquids must, as a result of thermal molecular motion, perform motions of such magnitude that they can easily be detected by a microscope”. He continues “It is possible that the motions to be discussed here are identical with the so-called “Brownian molecular motion”; however, the data available to me on the latter are so imprecise that I could not form a definite opinion on this matter”.

Robert Brown, the english botanist, had observed random motion of pollen grains in a liquid in 1828. The motion was analogous to a drunkard’s walk around a lamp post. Brown as a result of his experiments ruled out the possibility that the observed motion was due to pollen grains being moved by some vital force ie due to their living nature. Many different suggestions such as effect of capillarity, role of convection currents, evaporation, interaction with light, and electrical forces were put forward to explain these random motion. Even kinetic theory was proposed as a possible explanation but Von Nageli, in 1879, ruled it out for reasons which appeared cogent. He took straight segments on the path of a Brownian particle to be their free motion between two collisions with molecules. We now know that even these straight segments arises due to the effect of multiple collisions with atoms. In fact one of achievements of Einstein in this paper was to clarify the physically significant observations to make on these particles.

Einstein calculated the diffusion constant D for the suspended microscopic particles, of the size ‘a’ of the order of one-thousandath of a millimeter, in the liquid and showed that it is given by

$$D = RT/(6\pi\eta a N_A)$$

where T is the temperature, η the viscosity of the liquid. As before N_A is the Avogadro Number and a determination of D would provide us another method to measure it. One would recall that same formal expression for D had appeared in his thesis on molecular sizes. There it was however for solute molecules while here it is for suspended particles in the liquids. One would think that these two situations being analogous, except for the size of the diffusing particle, the same formula should be valid. However in those days it was not believed that Vant-Hoft’s law of osmotic pressure is applicable for both the solute molecules as well as for suspended particles. Einstein showed using molecular kinetic theory that it is indeed valid for both.

Einstein then showed that a measurement of mean square fluctuation in the x-component of position of a Brownian particle, $< x^2 >$ in time t provides us with a way to measure D as

$$\langle x^2 \rangle = 2Dt,$$

assuming that this prediction of $< x^2 > /t$ being constant is correct. That can however be always checked by the experiment. This is the first example of a fluctuation – dissipation theorem in physics.

5.3 Jean Perrin

J. Perrin, and his group, beginning 1908, carried out a series of beautiful experiment on the Brownian of colloid particles in suspension. They were able to produce the colloidal particles of a uniform size. Their work resulted in a complete confirmation of Einstein's results and a precise determination of the Avogadro's number. As a result of Perrin's work the atomism triumphed. Even the arch-disbeliever in atoms, F.W. Ostwald, was convinced of their existence. As he wrote in a new edition of his "Outline of Chemistry", published by the end of first world war, "I am now convinced that we have recently become possessed of experimental evidence of the discrete or grained nature of matter for which the atomic hypothesis sought in vain for hundreds and thousands of years". As Perrin, himself, wrote in his book Les Atomes (1913), "The atomic theory has triumphed. Until recently still numerous, its adversaries, at last overcome, now renounce their misgivings, which were for so long, both legitimate and undeniably useful". J. Perin was awarded the Nobel Prize for this work in 1926.

5.4 Further work

Einstein also gave another derivation of the diffusion equation in this paper based on treating the motion of a Brownian particle as, to use modern terminology, a random Markov process. This derivation is to be contrasted to the classical derivations which were based on continuum mechanics. He thus connected diffusion process of many particles, approximated as a continuum, to the random-walk problem of individual particles. Further developments in the theory of Brownian motion have resulted in great progress in the study of stochastic processes, fluctuation phenomenon thus giving birth to most of statistical physics. All these have their origins in the Brownian motion paper of Einstein.

6 Classical Physics and It's Discontents

The thesis and the Brownian motion paper of Einstein were of great importance to physics in view of their bearings on the "reality of molecules", as well as for other reasons mentioned earlier. They were however perfectly in the mold of classical physics. With his papers on special theory of relativity and on quanta, which we discuss later, he was launching revolutions in classical physics. To discuss these conceptual revolutions it is first necessary to give some idea of the conceptual structure of classical physics.

6.1 Classical physics

The foundations of classical Newtonian dynamics were laid by Isaac Newton in his annus mirabilis and published in his magnum opus “Principia”, or to give it its’ full title “Philosophiae Naturalis Principia Mathematica”, in 1687. Newtonian world consists of point particles which influence each other by mutual forces “acting at a distance from each other”. He also discovered the universal inverse square law of gravitation between masspoints, thus unifying physics and astronomy. The masspoints move with time in an arena of three dimensional space. Space and time are absolute in the sense that the motion of the particles does not affect them. Thus the drama of particle motion is played on an unchanging fixed stage of the space and time.

Newton also viewed light as a stream of discrete particles. Christian Huygen, was first to propose in 1678 that light is better described as a wave motion. Later discoveries of interference of light by Thomas Young in 1801, and of diffraction of light by Augustin Fresnel gave a strong support to the wave theory of light and it was firmly established. Since it was inconcievable in those days to think of wavemotion without a medium, which would oscillate and support it’s propagation, a universal medium “luminiferous aether” was postuated to exist.

The concept of continuous field, which pervades over space, like a magnetic field, unlike point particles of Newton was introduced by Michael Faraday around the middle of the nineteenth century. Clerk Maxwell achieved, in 1864, the synthesis of two disparate fields, electric and magnetic fields, into a coherent unified field “electromagnetic field” in which the two affected each other. A completely unexpected prediction of Maxwell’s equations was that of transverse electromagnetic waves. The velocity of these waves, now denoted by c, involved electrical and magnetic quantities. On calculation this velocity c was found to be the same as the known velocity of light. Maxwell therefore made the brilliant suggestion that light is the same entity as these electromagnetic waves. This ends over lightening review of classical physics as it was at the end of the nineteenth century.

6.2 Two clouds on the horizon

Lord Kelvin, in a very perceptive and insightful lecture before the Royal Institution in April 1900 talked about two “Nineteenth century clouds over the dynamical theory of heat and light”. One of these involved the continued unsuccessful attempts to experimentally measure the motion of the earth through luminiferrous aether. The other one of these referred to the failure of equipartition of energy in classical statistical mechanics.

Rest of the Einstein’s work during the miracule year 1905 is devoted to a dispelling of these two ominous clouds on the horizon over the classical physics. His papers on special theory of relativity deal with a resolution of “earth’s velocity through aether” puzzle. This involves a complete overhaul of classical concepts of space and time. His paper on the “light quantum” deals with other cloud and ushered in the quantum revolution. We now turn to these papers now.

7 Special Theory of Relativity

7.1 Galilean relativity

Newton's laws of motion are valid in a set of special frames of reference. These are called "inertial frames of reference". For example Newton's first law says that a mass point, not acted upon by any external force, keeps moving in a straight line with a uniform speed. Now a particle which is moving in such a fashion in an earth-laboratory will not appear to moving in a straight line when viewed from the Sun due to earth's daily rotation and it's annual revolution around the Sun. Clearly the two frames of reference, i.e. one in which the earth is at rest and other one in which the Sun is at rest, can not both be inertial frames of reference.

How do we know if some particular frame of reference is inertial? We first note that if a frame of reference S is inertial then any other frame of reference S' which is moving in a straight line with uniform velocity, is also inertial. This specifies the class of frames of reference which are inertial and in which Newton's three laws of motion hold. In order to characterise the class of the inertial frame we have to specify at least one of them. In practice for solar system applications, it was taken to be the frame in which the center of mass of the solar system is at rest or in uniform rectilinear motion. Within the accuracy required in these calculation, it was same as the one in which the center of mass of the universe was at rest of uniform linear motion or the one in which the system of fixed stars was at rest or uniform linear motion.

The rules for comparing the space and time coordinate measurement in different inertial frames are known as Galilean transformation. Newton's laws obey the principles of Galilean relativity. Their form is invariant, ie unchanged, under Galilean transformation between two inertial reference systems.

7.2 Maxwell's electromagnetic theory and Galilean relativity

Note that as long as Newton's laws of motion are the only fundamental laws of physics there is no way in which one can determine the absolute velocity of any inertial frame with respect to some absolutely fixed point at rest. This situation radically changes with the advent of Maxwell's equations for electromagnetism.

We note that Maxwell's equation do not have the same form in different inertial frames connected by Galilean transformations. That is they are not invariant under them. For example the velocity of electromagnetic waves (i.e. light) is a constant. One can ask, in which inertial frame is it so? Because it can not be so in all inertial frames, which are connected by Galilean relation. If it is given by $\vec{c}$ in its' direction of propagation S, it would be $\vec{c}+\vec{v}$ in the frame S' which is moving with a velocity $\vec{v}$ rectilinearly with respect to S. The velocity of light was thus a fixed constant c only in the frame in which the luminiferous aether is at rest.

Taking advantage of this clash, between invariance of Newton's laws and non-invariance of Maxwell's electromagnetic theory of light, thus opens a way by which the motion of earth, for example, can be experimentally measured with respect to universal aether. A large number of methods were thought for this purpose. All of them gave a null result. Experiments were unable to detect the motion of the earth through aether. Most celebrated and accurate experiment devised for this purpose was by Michelson and Morely in 1887, which also reported a null result. As Maxwell summarised in an article in Encyclopedia Britannica, “The whole question of the state of the luminiferous medium near the earth, and of it’s connection with gross matter, is very far as yet from being settled by experiment”.

7.3 Einstein’s Resolution: Special theory of Relativity

Einstein’s resolution of “earth-aether velocity” problem was obtained by a thorough revision of Newtonian concepts of absolute space and absolute time. In this revision he was guided by his analysis of the concept of simultaneity. If the two events take place in a single frame of reference, e.g. a railway platform or a uniformly moving railway train on linear tracks, there is no difficulty in saying whether the two events are simultaneous in the same single frame of reference. If you start thinking about the problem one finds that the two events which look simultaneous in one frame, say railway platform, are not so in another relatively moving frame, e.g. that of a moving train. This is because the light signals used to observe the two events, whose simultaneity we are discussing, will take different times in the frames of two relatively moving observers. This is due to speed of light signal being finite. Since simultaneity is not an invariant concept it follows that time can not be absolute.

Einstein wished to hold on to what is now known as the two postulates of his “special relativity theory”. These are

(i) All physical laws have the same form in all inertial frames i.e. frames of references which move rectilinearly with a constant velocity with respect to each other;

(ii) The velocity of light is same in all inertial frames.

These two postulates look irreconcilable within Newtonian notions of absolute space and absolute time and Galilean transformation. However if Newtonian space and time concepts are modified so as to be in accord with what one has learnt from Einstein’s analysis of ‘simultaneity’, then it was Einstein’s insight that the two postulates considered above can indeed be reconciled. They can then support a new structure of space and time, now called space-time. As Minkowski said in 1908 in a lecture given at Cologne in 1908 “Hereafter space and time are bound to fade away and only a union of the two will preserve an independent reality”.

It is clear since time is not invariant in different inertial frames we can not maintain the Galilean transformations as the proper ones to connect them. Einstein goes on to show that they have to replaced by Lorentz transformations. This results in providing purely kinematic derivation of Fitzgerald-Lorentz length contraction which had been postulated earlier to explain null aether-earth velocity results. It also leads to Einstein time dilation i.e. moving objects live longer.

Under Lorentz transformations Maxwell’s theory, it is satisfying to note, is invariant. Newtonian dynamics, being invariant under Galilean transformation, is however not so and it has therefore to be modified. These result in mass variation with velocity. All these phenomenon are observed in high energy accelerators on a routine basis.

Einstein was asked to donate his original manuscript of “special relativity paper” for Kansas War-bond rally for auction. Since he did not have it any more he copied out the published version in long-hand. He did not however make any corrections despite feeling that he could have phrased it better in many place. It went for six and a half million dollars at the auction and was deposited in the Library of Congress at Washington.

7.4 $E = mc^2$

In his special relativity paper Einstein missed out on a profound result which led to energy-mass equivalence. This was published separately in a 3 page. Befor this paper one had two seperate conservation laws, one for the mass and another for energy, in physical transformations. With this insight they merged into only one conservation law.

He noted that "The mass of a body is a measure of its energy contrast; If the energy changes by E, the mass changes in the same sense by E/c^2, $\cdots$". We have modified the notation in this translation to accord with modern usage. The published paper has L in place of E and "$L/9.10^{20}$, if the energy is measured in ergs and mass in grams" in place of modern E/c^2.

He also noted "Perhaps it will prove possible to test this theory using bodies whose energy content is variable to high degree (e.g. salts of radium)". This was quite prophetic in view of its' eventual sad use in nuclear explosions. Coming back to his letter to Habicht, the Lord was indeed chuckling and leading him by the nose.

Incidently the equation $E = mc^2$ is the only equation which occurs in Bartlett's familier quotation. It has acquired in our culture an iconic status and is seen on bill boards, T-shirts and so on.

7.5 Einstein's later related work

We just give a chronological listing of important land marks.
1907: Discovery of Principle of Equivalence
1912-13: Gave metric tensor description of gravitation
1915, Nov. 25: Completes his formulation of General theory of Relativity. Space-time which was so far regarded as flat and Euclidean is now modified to a curved Riemannian space time. The curvature of space-time is identified with gravitation.
1917: First paper on Cosmology
1919, May 29: The total solar eclipse expedition led by British astronomer Eddington confirms General theory of Relativity of German Einstein. This provides a shining example of international peaceful scientific collaboration between scientists, even though belonging to enemy nations in just concluded First World War. Einstein becomes a World-icon.
After 1922: Einstein works unsuccessfully on unifying electromagnetsm with gravitation.

We also wish to note down about the first ever English translation of Einstein and Minkowski's papers from German was published by Calcutta University in 1920. The translators were M.N. Saha and S.N. Bose and a historical introduction was provided by P.C. Mahalarobis.

8 Quantum Revolution

We now discuss his light quantum paper which was his most revolutionary one in 1905.

8.1 The Black Body Radiation: Kirchhoff to Planck

The origins of the quantum revolution are in the problem of Black-Body Radiation. All heated bodies emit radiation energy as well as absorb it. A consideration of thermodynamic equilibrium led Gustov Kirchhoff of Berlin, in 1859 to conclude that the ratio of emissivity to absorptivity of the radiation does not depend on the nature of the heated body. This ratio, a universal function is the same as the emissivity of a perfect black body i.e. a body which completely absorbs all the radiation falling on it. It was also shown that the radiation inside a heated cavity is same as black body radiation. Max Planck occupied Kirchhoff's chair at Berlin in 1889. He argued that as the ratio is independent of the nature of the cavity material he should be able to calculate it by using a simple model for the material of the cavity. The model he used was that it is made of Hertzian oscillators each with a single frequency ν. Using this model he could show that the universal function is related to average energy of each oscillator of frequency ν at the temperature T of the Black Body radiation. He had this result on 18 May 1899.

If Planck had known the equipartition theorem of classical statistical mechanics, for average energy, at this point, he would have obtained the law of Black body radiation now known as Rayleigh-Jeans radiation law as it was given by Rayleigh in June 1900 and corrected for a missing factor of 8 by Jeans in June 1905. Indeed this was first done by Einstein in his light quantum paper. Amusingly he did it before Jeans. Rayleigh-Jeans radiation law was found applicable only at small values of ν/T and not for large values of ν/T. One thus became aware of the second cloud on the horizon of classical physics referred to by Lord Kelvin viz the failure of equipartition of energy.

Guided by the precision experimental results on black body radiation Planck announced an empirical radiation law on Oct.19, 1900 which fitted the data perfectly. The Planck's radiation law is now known to be the correct law of black body radiation. It had the same form as Rayleigh's law for small ν/T and the form of empirically proposed Wien's law, given in 1894, for large ν/T. Planck however had no theoretical basis for his radiation law.

Planck next presented a derivation of his radiation law. He was so desperate that he even used Boltzmann's probability interpretation for entropy. The derivation was announced to German Physical Society on Dec. 14, 1900. The really new element in his derivation was his assumption that a Hertzian oscillator, of frequency ν, can emit or absorb radiation only in integral multiples of a basic quantum of energy ϵ, where $\epsilon = h\nu$. The constant h is now known as Planck's constant. In classical physics there was no such discreteness. The oscillator could emit or absorb radiation of any energy. This was the first parting of ways with classical physics. Planck however took this assumption as a purely a formal one and did not quite realise that something radical has been introduced. As he said "This was a purely formal assumption and I really did not give it much thought except that no matter what the cost, I must bring out a positive result".

8.2 Einstein's light quantum hypothesis

Einstein was the first person to realise that Planck's introduction of energy quanta was a revolutionary step. As we noted a while ago Einstein, in his light quantum paper, first showed that the so called 'Rayleigh-Jean's Law' is the unambiguous prediction of classical physics for the radiation law. This law not only does not work for high frequency radiation, it also theoretically suffers from 'ultraviolet catastrophy' (i.e. infinite energy). This convinced Einstein that to get the correct radiation law, a break with classical physics is involved.

In his quest for the cause of the failure of classical physics Einstein was guided by his unhappiness with asymmetrical treatment of matter and radiation in classical physics. Matter is discrete and particulate while radiation is continnuous and wave like. He thus proposes that radiation is also particle-like just as matter is, i.e. his "light quantum" hypothesis, and is not wave like. He was of course fully aware of the successes of wave theory in dealing with the phenomenon of interference and diffraction of light. All these phenomenon, however, need only time averages and for such phenomenon wave theory probably is indispensible. It is however concievable that a basic particle picture on time averaging could produce wave like behavior. He sumarised that the particle nature of radiation may show up in the processes involving the generation and transformation of light where we deal with instantaneous processes.

Can one adduce any evidence in favour of particle nature of light? Einstein proceeds to show that a consideration of Wien's radiation law, valid in the nonclassical regime of large frequencies, does that. He calculates the probability p that the monochramatic radiation of frequency ν, occupying a volume V_0, could be confined to smaller volume V using Wien's law. The result is

$$p = (V/V_0)^n \text{ with } n = E/h\nu$$

where E is the total energy of the radiation. This is of the same form as for a gas of n particles. From this remarkable similarity, Einstein concludes "Monochromatic radiation of low density (within the range of validity of Wien's radiation formula) behaves thermodynamically as if it consisted on mutually independent energy quanta of magnitude $R\beta\nu/N$". (In modern notation $R\beta\nu/N$ reads as $h\nu$). This is the Einstein's light quantum hypothesis. In this picutre "the energy of light is discontinuously distributed in space. $\cdots$ when a light ray is spreading from a point is not distributed continuously over ever increasing spaces, but consists of a finite number of energy quanta that are localised in points in space, move without dividing, and can be absorbed or generated only as a whole".

Einstein applied successfully his light quantum hypothesis to other phenomenon involving generation and transformation of light. The most important of these was his treatment of photoelectric effect. He also discussed Stokes' rule in photoluminescence and ionisation of gases by ultraviolet light.

8.3 The photoelectric effect

In 1887 Heinrich Hertz observed that the ultraviolet light incident on metals can cause electric sparks. In 1989 J.J. Thomson established that the sparks are due to emission of the electrons. Phillip Lenard showed in 1902 that this phenomenon, now called the Photoelectric effect, showed 'not the slightest dependence on the light intensity' even when it was varied a thousandfold. He also made a qualitative observation that photoelectron energies increased with the increasing light frequency. The observations of Lenard were hard to explain on the basis of electromagnetic wave theory of light. The wave theory would predict an increase in photoelectron energy with increasing incident light intensity and no effect due to increase of frequency of incident light.

On the Einstein's light quantum picture, a light quantum, with energy $h\nu$, on colliding with an electron in the metal, gives its entire energy to it. An electron from the interior of a metal has to do some work, W, to escape from the interior to the surface. We therefore get the Einstein photoelectric equation, for the energy of the electron E,

$$E = h\nu - W.$$

Of course electron may lose some energy to other atoms before escaping to the surface, so this expression gives only the maximum of photo-electron energy which would be observed. One can see that Einstein's light quantum picture explains quite naturally the intensity independence of photoelectron energies and gives a precise quantitative prediction for its dependence on incident light frequency. It also predicts that no photoelectrons would be observed if $\nu < \nu_0$ where $h\nu_0 = W$. The effect of increasing light intensity should be an increase in the number of emitted electrons and not on their energy. Abram Pais has called this equation as the second coming of the Planck's constant.

Robert A. Millikan spent some ten years testing Einstein equation and he did the most exacting experiments. He summarized his conclusions as well as his personal dislike of light quantum concept, as follows: 'Einstein's photoelectric equation $\cdots$ appears in every case to predict exactly the observed results $\cdots$ yet the semi-corpuscular theory by which Einstein arrived at his equations seems at present wholly untenable' (1915) and 'the bold, not to say reckless hypothesis of electromagnetic light corpuscle' (1916).

8.4 Envoi

Einstein was awarded Nobel Prize in Physics for 1921 for this paper on light quanta and especially it's application to the photoelectric effect. Even though his status as public icon is associated with his relativity theory, he was not awarded Nobel Prize, for that. He however delivered his Nobel Lecture on Relativity.

Einstein's light quantum was renamed as "photon" by G.N. Lewis as late as 1926. Though Einstein talked about photon energy, $E = h\nu$, in 1905, it is curious that he introduced the concept of photon momentum, $p = \frac{h\nu}{c}$, only in 1917. As we have seen even Millikan did not believe in photons around 1915-16 despite his detailed experimental work on photoelectric effect. In 1923, the kinematics of the Compton effect was worked out on the basis of it's being an elastic electron-photon scattering by A.H. Compton sucessfully. After that it was widely accepted that light does sometimes behaves as photon.

Einstein made the first application of quantum ideas to matter in his work on specific heat of solids in 1907. A consideration of energy fluctuations, using Planck's radiation law, led him to the dual particle-wave nature of radiation in 1909. In 1916-

17, in the course of a new derivation of the Planck's radiation law, using chemical kinetics methods, Einstein discovered the phenomenon of stimulated emission of light and introduced his famous A and B coefficients. These are of fundamental importance in the theory of lasers.

S.N. Bose in 1924 at Dacca sent Einstein a new derivation of Planck's law in which only the photon concept was used. Albeit the photons did not obey the classical statistics of Maxwell and Boltzmann, but rather a new statistics. Einstein saw the importance of this contribution, translated the paper into German, and got it published. He also applied it to the matter. The new statistics is now known as either Bose-Statistics or as Bose-Einstein statistics. The particles which obey this statistics are known as bosons. As a consequence of these statistical considerations Einstein discovered that a free gas of Boson undergoes a phase transition, Bose-Einstein condensation, below a critical temperature. The phenomenon was seen only in 1995 and a Nobel Prize awarded for it in 2001.

The modern mathematical formulation of quantum mechanics was obtained by W. Heisenberg and E. Schrodinger in 1925-26 in two different, but equivalent, forms. Einstein's role in achieving this transition from old quantum theory to the modern quantum mechanics was quite significant. He has been called godfather of Schrodinger's wave mechanics and his relativity theory with it's emphasis on operational procedure provided the inspiration to Heisenberg in his matrix mechanics.

After 1926 Einstein's focus shifted to foundational questions of quantum mechanics. He gave his ensemble interpretation of quantum mechanics. His discovery of nonlocal correlations in quantum mechanics with Podolsky and Rosen in 1935 was of far reaching significance and continues to spawn new fields, such as quantum computing, quantum information theory and quantum cryplography, down to the present time.

9 Epilogue

Einstein purchased in 1935 a white simple frame house at 112, Mercer Street within walking distance of his office at Institute for Advanced Princeton and he lived here till the end. There were a few pictures and etchings. These included a drawing of Gandhi, whom he admired greatly. There were photographs of his mother and his sister Maja who lived with him after she moved to Princeton from Italy in 1939. He had also brought with him, from Europe, three etchings of the physicist he admired more than any other. These were of Newton, Maxwell and Faraday. It is now given his contributions to physics abandantly clear that he himself belong to this select company.

When Einstein died the famous cartoonist Herblock published in Washington Post, a cartoon, in which the planet earth is identified by the words "Albert Einstein lived here". He became a world icon in 1919 and since then he has continued to hold a place of high esteem in public mind for his science, his humanity, his fight for a peaceful world and his freedom from cant. At the end of millenium He was voted by Time magazine, and many others as the "Man of the Millenium".

10 Bibiliographical Notes:

1. The literature on science and life of Einstein is enormous. The best biography for physicists is Pais, A., Subtle is the Lord $\cdots$: The Science and Life of Albert Einstein, Clarendon Press, Oxford and Oxford University Press, New York, 1982.
   A volume for a more general reader is Bernstein, J., Einstein, The Viking Press, New York, 1973.

2. For the writings of Einstein, we have the multivolume ongoing series,
   The Collected Papers of Albert Einstein,
   Princeton University Press, Princeton, N.J., 1987 $- \cdots$, and the companion volumes
   The Collected Papers of Albert Einstein: English Translation,
   Princeton University Press, Princeton, N.J., 1987 $- \cdots$. The brief quotations from Einstein's papers and letters and thesis reports used in this lecture are from Vol.2: The Swiss Years: Writings, 1900-1909; Vol.5: The Swiss Years: correspondence, 1902-1914 of this series of translation volumes. Einstein's papers from the miracle year 1905 are also available in English translation in Einstein's Miraculous Year: Five Papers that changed the Face of Physics, (ed. J. Stachel), Princeton, 1998. (Indian reprint by Srishti Publishers, New Delhi, 2001).
   There are also a number of other english translations of individual papers.

3. The quote from Isaac Newton is from R.S. Westfall,
   Never at Rest, A Biography of Isaac Newton, Cambridge, 1981.

4. On Einstein's doctoral thesis, see also N. Straumann: arXiv: physics/0504201 (April 2005).

5. For an appreciation of Einstein's contribution to the theory of random processes, see L. Cohen, The History of Noise, IEEE Signal Processing Magazine, p.20-45, Nov. 2005.

6. Ostwald's quote on the reality of atoms is from Bernstein's book (p.186) referred earlier.

7. For a more detailed writeup on Einstein's contributions to quantum theory, see V. Singh: Einstein and the Quantum, Current Science B9, 2101-2112 (2005). [There is some overlap of the present writeup with this paper]

8. For the impact of Einstein's work on the Physics of the twentieth century, see Physics World, Special Einstein issue, Jan. 2005; Current Science, Special Einstein Issue, (25 Dec. 2005).
   D. Giulini and N. Straumann, Stud. Hist. Phil. Mod. Phys., 37, 115-173 (2006).